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Optimal design of series-parallel systems considering maintenance and salvage value

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Abstract

A reliability based design (RBD) model is developed for a series-parallel system with deteriorating components in order to minimize the life cycle cost of the system. The effects of fixed asset depreciation, preventive maintenance and minimal repair are incorporated in the model. We also propose equations to model the effects of preventive maintenance on the system's failure rate and the salvage value as functions of time. Genetic algorithms (GAs) are used to perform constrained optimization of the system cost function subject to both active and non-active constraints. The results are useful for engineering economists, reliability engineers, and system designers.

Keywords: Reliability based design; Design for life cycle cost; Salvage value; Preventive maintenance; Minimal repair; Genetic algorithms

1. Introduction

The reliable performance of a system used is of utmost importance in many industrial, military, and everyday life situations. There has been an increasing need for systems and components with higher reliability and lower cost. There exist several methods for designing such systems. These methods include using large safety factors, reducing the complexity of the system, increasing the reliability of constituent components through a product improvement program, using structural redundancy, and practicing a planned maintenance and repair schedule. Much literature exists on these methods (Kececioglu, 1991).

A series-parallel system comprises of n subsystem in series while each subsystem consists of $(1 + m_j)$,

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Nomenclature

Notation

AAC_i	average annual cost of the system at the end of the ith interval
AC_j	acquisition cost of a component in subsystem j
$c_{\mathrm{m}j}$	cost of minimal repair for subsystem j
x_i	length of preventive maintenance interval i
$C_{\mathrm{M}ji}$	expected minimal repair cost of subsystem j during $(0,x_i]$
IC	installation cost of the system
$h_{ji}(t)$	hazard/failure rate function of a component in subsystem j during the ith interval
$H_{ji}(t)$	cumulative hazard/failure rate function of a component in subsystem j during the ith interval
$(1+m_j)$	total number of parallel components in subsystem j
MC_j	preventive maintenance cost of a component in subsystem j
PM_{ji}	the total cost of the first $i-1$ preventive maintenances for subsystem j
$\mathrm{SV}_{ji}(t)$	salvage value of a component in subsystem j at time t during ith interval
$\mathrm{NAC}_{ji}(t)$	net acquisition cost of a component in subsystem j at time t
Q_j, S_j, P_j	user defined constants based on component deterioration
$r_{ji}(t)$	reliability function of a component in subsystem j during the ith interval
$R_{ji}(t)$	reliability function of subsystem j during the ith interval
ξ	maximum allowed failure rate of the system
θ_{ji}	failure rate deterioration factor of a component in subsystem j during the ith interval
ϕ_j	assembly coefficient of a component in subsystem j
Γ_i	user defined constant (market driven), where $0<\Gamma_i<\Gamma_{i+1}$
ρ	user defined constant (market driven), where $\rho>0$
β	user defined constant (market driven), where $\beta\geq 0$

where, $j=1$ to n, identical components in active redundancy (Kececioglu, 1991). A subsystem is failed if all the components in the subsystem are failed. Failure of any subsystem causes the failure of the whole system. A traditional optimal redundancy allocation problem for such a system ignores that reliability is a function of time and simply involves calculating the optimal m_j for minimum system cost or maximum system reliability, while satisfying constraints such as weight, volume, costs, and/or reliability. Much research in the area of optimal redundancy allocation for series-parallel system has been done with the assumption that a component has a constant reliability. A detailed literature review related to optimization of series-parallel systems using components with constant reliability is presented in Tillman, Hwang and Kuo (1985) and Coit and Smith (1996).

In recent years, the research on *minimal repair* and *preventive maintenance* for equipment in use to maintain system reliability has become prevalent. A *minimal repair* of the failed system restores the function of the system in such a way that its failure rate remains as it was just before the failure. On the other hand, *preventive maintenance* (PM) categorizes those actions which improve the condition of the system before the system fails. Conventional PM policies assume that the system after each PM intervention is restored as new. However, this assumption does not hold true in most of the situations as the state of the system after a PM action is somewhere between as good as new and as bad as old. A PM action which does not return the system to its original condition is also referred to as *imperfect repair* (Jayabalan & Chaudhari, 1992).

Maintenance policies for systems with increasing failure rates, considering minimal repair and/or preventive maintenance have been studied by many researchers. A policy of periodic replacement with minimal repair at failure for systems with an increasing failure rate is one in which the system is replaced at multiples of some period T while minimal repair is performed at any intervening systems failures. This type of policy was introduced and investigated by Barlow and Hunter (1960) and further generalized and modified by Barlow and Proschan (1965), Tilquin and Cleroux (1975), Nakagawa (1981) and Boland and Proschan (1982). An extensive survey of the minimal repair policies is available in a paper by Valdez-Flores and Feldman (1989).

In a policy of system replacement with PM, systems which have to work at or below a maximum acceptable failure rate are maintained preventively at points such that the system does not exceed the acceptable failure rate. As the system ages, the frequency of PM actions increases and the system is replaced when it is not economically justified to keep the system (Jayabalan & Chaudhari, 1992). Many authors have modeled the effects of PM on existing systems (Brown & Proschan, 1983; Canfield, 1986; Jayabalan & Chaudhari, 1992; Malik, 1986; Murthy & Nguyen, 1981; Nakagawa, 1986 Nakagawa, 1988).

The concepts of minimal repair and PM can be incorporated into reliability based design (RBD) if the component reliability is expressed as a function of time. Monga, Zuo and Toogood (1995a,b) combined research works on RBD with constant component reliabilities and on maintenance models wherein time is an important factor. They reported RBD models for series-parallel systems in which the reliability of the components is a function of time. In this paper we model systems which have a non-zero failure rate at time zero due to externally and internally induced conditions. The acquisition cost of the system is adjusted to incorporate the effects of asset depreciation at the time of disposal. Genetic Algorithms (GAs) are used to minimize the expected average annual costs of the system. Section 2 discusses the system characteristics, costs and details of the system maintenance policy. Section 3 presents a brief overview of GAs and its advantages over other optimization methods. Section 4 presents numerical solutions of an example followed by discussion of results and concluding remarks.

2. System characteristics and problem formulation

In this paper, we consider a system which comprises of n subsystems in series. Subsystem $j(j = 1, \ldots, n)$ consists of $(1 + m_j)$ identical components in active redundancy. The lives of the components are random variables and are assumed to be statistically independent of one another. All the components have continuous and strictly increasing failure rates. It is required that the system should operate below a maximum allowed failure rate.

For the systems with increasing failure rates it is a common practice to perform PM whenever the system reaches the maximum allowed failure rate or minimum acceptable level of reliability (Jayabalan & Chaudhari, 1992; Lie & Chun, 1986; Malik, 1986). If the system fails in between these PM intervals, then minimal repairs are performed. Such a policy was proposed by Nakagawa (1988) and further validated by Jayabalan and Chaudhari (1992) with a case study. Nakagawa (1988) presented two sequential PM models using *age reduction* and *hazard rate concepts*. According to Nakagawa's *hazard*

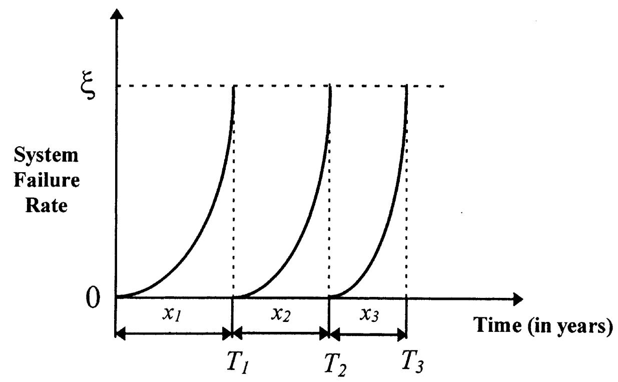

Fig. 1. System failure rate and preventive maintenance intervals (hazard rate concept, Nakagawa, 1988).

rate model, a PM restores the system to a working condition with a failure rate of zero and a reliability of one. However, after each additional PM action, the slope of the hazard function increases. As shown in Fig. 1, the PM interval x_i decreases as i increases in Nakagawa's hazard rate model. In general, the hazard rate of a component in the jth subsystem during the ith interval is $h_{ji}(t)$:

$$h_{ji}(t) = \theta_{ji} h_j(t) \qquad \text{for} \qquad i \geq 1, \tag{1}$$

where $h_j(t)$ is the component's hazard rate function when it has not gone through any PM and θ_{ji} is defined as *failure rate deterioration factor*. Depending on the effect of a PM action on the component, one can mathematically define θ_{ji} to satisfy the following two conditions (Monga, Zuo & Toogood 1997):

1. $\theta_{j1} = 1$
2. $\theta_{j(i+1)} \geq \theta_{ji}, \qquad where \qquad i = 1, 2, 3...$

In many practical situations the failure rate of the system is non-zero after each PM action due to both externally and internally induced conditions. To model this more practical condition we modify the failure rate function (1) by adding a constant λ_j to variable t, such that $\lambda_j > 0$. During the first interval ($i = 1$)

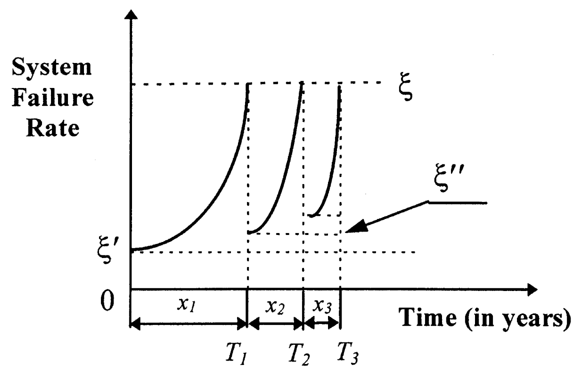

Fig. 2. System failure rate and preventive maintenance intervals (using the modified PM model).

when the component has undergone no PM, the component failure rate is $h_{j1}(t)$ where $0 \leq t \leq x_1$ (Fig. 2). This failure rate function corresponds to the original failure rate of the component, i.e. $h_{j1}(t) = h_j(\lambda_j + t)$. At $t = 0, h_{j1}(0) = \xi' = h_j(\lambda_j) > 0$, After the component undergoes PM at the end of the first interval, its failure rate is now changed to $h_{j2}(0) = \theta_{j2} h_j(\lambda_j) = \xi'' > \xi'$ (Fig. 2). In general, the modified hazard rate function of a component in the jth subsystem during the ith interval can now be rewritten as:

$$h_{ji}(t) = \theta_{ji} h_j(\lambda_j + t) \tag{2}$$

Nakagawa (1988) provides a mathematical expression for hazard rate deterioration factor θ_{ji} as

$$\theta_{ji} = \prod_{k=0}^{i-1} \left(1 + \frac{k}{k+1}\right). \tag{3}$$

According to his proposed formula, the system deteriorates drastically as the number of PM actions increase. This expression is suitable for systems for which PM actions do not improve the condition of the system considerably. However, for many mechanical systems, PM actions like cleaning, lubrication, adjustment and alignment can improve the condition of the system significantly. To model such an action the change in the deterioration factor should increase gradually as the number of PM actions increases. We give the following general equation for calculating θ_{ji}.

$$\theta_{ji} = 1 + \sum_{k=0}^{i-1} \left(\frac{Q_j k}{S_i k + P_j} \right) \tag{4}$$

where Q_j, S_j and P_j are user defined constants and greater than zero. These constants can be defined by the user based on component deterioration characteristics before and after the PM action. A large value of Q_j and relatively smaller values of S_j and P_j will be appropriate for very fast deteriorating system where PM action would make the system operational but would not be able to prevent it from deteriorating drastically after each PM action. On the other hand to model the effect of PM action on mechanical systems, the user is suggested to use a lower value of Q_j, and relatively larger values of S_j and P_j.

Similar to Eq. (2), one can write the following equation for the cumulative hazard function

$$H_{ji}(t) = \theta_{ji} H_j(\lambda_j + t), \qquad 0 \leq t \leq x_i. \tag{5}$$

Next we rewrite the system hazard rate in terms of component hazard rate. This will allow us to incorporate different deterioration factors for different subsystems at the component level. The reliability of a component in subsystem j during the ith interval is

$$r_{ji}(x_i) = \mathrm{e}^{-\int_0^{x_i} h_{ji}(t)\,\mathrm{d}t} = \mathrm{e}^{-H_{ji}(x_i)} \tag{6}$$

where x_i is the length of interval i. The system reliability can be written as

$$R_{si}(x_i) = \prod_{j=1}^{n} [1 - [1 - r_{ji}(x_i)]^{(1+m_j)}] \tag{7}$$

while the corresponding system hazard rate is

$$h_{si}(x_i) = \frac{-R'_{si}(x_i)}{R_{si}(x_i)} \tag{8}$$

PM is performed exactly when the system reaches the maximum allowable failure rate. Minimal repairs are performed if the system fails before the maximum allowable failure rate is reached. Fig. 2 shows the PM scheduling and the system failure rate behavior after each PM action. The system should always operate below a maximum allowable failure rate, which implies that Eq. (9) should always be satisfied

$$h_{si}(t) \le \xi, \qquad 0 \le t \le x_i. \tag{9}$$

There are four categories of system costs in our formulation. They are: (a) *net acquisition cost* of the system, (b) *installation cost* of the system, (c) *cost due to minimal repairs*, and, (d) *cost of performing preventive maintenance*. The acquisition cost includes design, development and production costs of the system. The *net acquisition cost* of a component at a given time t incorporates both acquisition cost and salvage value. We propose to use Eq. (10) to represent the net acquisition cost.

$$\mathrm{NAC}_{ji}(t) = \phi_j(1 + m_j)[\mathrm{AC}_{ji} - \mathrm{SV}_{ji}(t)], \tag{10}$$

where assembly coefficient ϕ_j accounts for the assembly cost of a component in the jth subsystem and SV_{ji} is the salvage value function of the component in the jth subsystem during the ith interval.

The salvage value is defined as market value of a component/system at the end of its life. It is the amount eventually recovered through sale, trade-in or salvage (Park, Porteous, Sadler & Zuo, 1995). The salvage value of the system is estimated from a depreciation schedule established for a system. Depreciation can be classified into two categories, namely, physical and functional depreciation. Physical depreciation is defined as a reduction in a system's capacity to perform its intended service due to physical impairment. Physical depreciation can occur to any fixed asset in the form of deterioration from interaction with the environment, including such agents as corrosion, rotting, etc. It can also occur due to system wear and use. Physical depreciation leads to decline in performance and high maintenance costs. Functional depreciation occurs as a result of changes in the organization or in technology that decrease or eliminate the need for a system. Examples of functional depreciation include obsolescence due to advances in technology, a declining need for the services performed by a system, or the inability to meet increased quantity and/or quality demands.

Many researchers have used the concept of economic depreciation in various areas of research related to evaluation of manufacturing systems. Kaio and Osaki (1990) have used the economic depreciation to evaluate a one unit system, where each failed unit is scrapped without repair and each spare is provided through an order with a lead time. In their models they consider a non-linear increasing cost $s(t)$, which is suffered by the system for salvage at time t. Jones, Zydiak and Hopp (1990) present an equipment replacement model for profit maximizing firm facing a demand curve. The firm's product is produced by machines whose capacity is a non-increasing function of age. The economic depreciation schedule is developed using the concepts from microeconomics. The objective of the firm is to maximize the sum of discounted profits over an infinite period. Gupta and Dai (1991) used the concept of salvage value for perishable product retailing systems like grocery and fashion clothing. The authors for an assumed salvage value calculated the maximum profit ordering. Cheng (1992) analyzed an optimal replacement problem of an aging equipment, where he defines overhaul and inspection costs as an increasing function of the inspection interval to model the equipment depreciation. The replacement cost is a decreasing function of the inspection interval. Total cost per unit time is minimized which minimizes the total cost of replacements and operating the equipment between the overhaul and inspection intervals.

In this paper, we propose a salvage value function which incorporates the economic effects of system deterioration and PM. The proposed salvage value function satisfies the following three conditions.

1. The salvage value function of a system with an increasing failure rate is a monotonically decreasing function during any given operational interval.
2. At the end of any interval, if a PM action is performed on the system then the salvage value of the system increases to a value such that

$$\mathrm{SV}_{ji}(t = x_i) < \mathrm{SV}_{j(i+1)}(t = 0) < \mathrm{SV}_{ji}(t = 0) \tag{11}$$

3. Functional depreciation of the system and the tax effects of system disposal are ignored.

To satisfy the above three conditions, we propose the following salvage value function of a component in subsystem j during the ith interval

$$\mathrm{SV}_{ji}(t) = \frac{\mathrm{AC}_j}{\Gamma_i[\rho h_{ji}(t) + \beta]^t} \tag{12}$$

where Γ_i, ρ and β are the user defined constants which are market driven such that $0 < \Gamma_i < \Gamma_{i+1}$, $\rho > 0$ and $\beta \geq 0$. It will be shown through numerical illustration that $\mathrm{NAC}_{ji}(t)$ plays an important role in system replacement decision making as it is an increasing cost function with respect to time. The *installation cost* (IC) of the system is a one time cost during system life cycle and is independent of system design. The costs associated with minimal repairs depend on the frequency of failures in a system. The frequency of these failures is related to the individual subsystem's failure rate function. Consider subsystem j in the time interval $[0,x_i]$. On failure, minimal repairs are performed at cost c_{mj}. According to theorem 1 by Boland (1982), the expected minimal repair cost of subsystem j in interval $[0,x_i]$ is C_{Mji}

$$C_{Mji} = \int_0^{x_i} c_{mj} h_{ji}(t)\,dt \tag{13}$$

The expected minimal repair cost of subsystem j at the end of the first interval is given by

$$C_{Mj1} = c_{mj} \int_0^{x_1} h_{j1}(t)\,dt$$

$$= c_{mj}\theta_{j1} \int_0^{x_1} h_j(\lambda_j + t)\,dt \tag{14}$$

Since $\theta_{j1} = 1$ hence we can rewrite Eq. (14) as

$$C_{Mj1} = c_{mj}[H_j(\lambda_j + x_1) - H_j(\lambda_j)] \tag{15}$$

The number of minimal repairs at the end of the second interval is given by

$$C_{Mj2} = C_{Mj1} + c_{mj}\int_0^{x_2} h_{j2}(t)dt \tag{16}$$

$$= C_{Mj1} + c_{mj}\theta_{j2}\int_0^{x_2} h_j(\lambda_j + t)\,\mathrm{d}t$$

$$= C_{Mj1} + c_{mj}\theta_{j2}[H_j(\lambda_j + x_2) - H_j(\lambda_j)]$$

$$= c_{mj}[H_j(\lambda_j + x_1) - H_j(\lambda_j)] + \theta_{j2}[H_j(\lambda_j + x_2) - H_j(\lambda_j)]$$

$$= c_{mj}\sum_{k=1}^{2}\theta_{jk}[H_j(\lambda_j + x_k) - H_j(\lambda_j)]$$

The minimal repair cost of subsystem j at the end of i intervals can be given by Eq. (17)

$$C_{Mji} = c_{mj}\sum_{k=1}^{i}\theta_{jk}[H_j(\lambda_j + x_k) - H_j(\lambda_j)] \tag{17}$$

If PM is performed on the system then all the components undergo PM. Thus the cost of a PM action for subsystem j with $(m_j + 1)$ components will be $\mathrm{MC}_j(m_j + 1)$, where MC_j is the cost of performing PM on a component in subsystem j. The cost of PM on subsystem j at the end of the ith interval is given by

$$\mathrm{PM}_{ji} = (i - 1)\mathrm{MC}_j(m_j + 1) \tag{18}$$

When the system reaches the maximum allowed failure rate, one of the following decisions is made: (1) keep the system and perform PM, or (2) replace the system with an identical system. This decision is made by comparing the expected average annual costs (AAC) of the system and checking whether the system has reached its economic life or not. The AAC of the system at the end of the ith interval, given PM at intervals x_1, x_2, x_3,… is given by Eq. (19)

$$\mathrm{AAC}_i = \frac{\mathrm{IC} + \sum_{j=1}^{n}[\mathrm{NAC}_{ji}(t) + \mathrm{PM}_{ji} + \mathrm{C}_{Mji}]}{T_i} \tag{19}$$

where

$$T_i = \sum_{k=1}^{i} x_k \tag{20}$$

If the values of m_j are known, then one can use Eqs. (9), (19) and (20) to calculate the values of T_1, T_2,…,T_i, T_{i+1} and subsequently calculate the values of AAC_1, AAC_2,…, AAC_i, AAC_{i+1}. We stop when $\mathrm{AAC}_{i+1} > \mathrm{AAC}_i$ and conclude that T_i is the economic life of the system and the system is replaced at T_i. However, since m_j is unknown, we cannot calculate the PM intervals, AAC_j or replacement time of the system directly. In addition, traditional RBD problems have constraints of weight, cost and volume. These constraints unlike the failure rate constraint (9) are not necessarily active.

To calculate the optimal system life cycle, its PM intervals, and system replacement time, procedure proposed by Monga et al. (1995, 1997) is used. This procedure provides the user with optimal system design over a system's life cycle. To calculate optimal values of m_j we minimize AAC subject to failure rate and resource constraints for each value of i to obtain AAC_i^*. Once the optimal system configuration is obtained for the first i intervals, we calculate AAC_{i+1} using the optimal configuration obtained for i. If this AAC_{i+1} is less than AAC_i^* then it implies that for the given optimal configuration for intervals 1 to i, the corresponding value of T_i is not the system's economic life. The system design is then optimized for $i+1$ intervals. This process is repeated and we stop when AAC_{i+1} is greater than AAC_i^*. The basis behind this methodology is that if the system is replaced after the ith interval, the system cost should be minimum for T_i. The procedure for this methodology is listed below:

Step 1. Set $i = 1$
Step 2. Minimize the expected average annual costs AAC subject to failure rate and resource constraints. The optimization problem is formulated as:
Minimize: AAC_i, subject to:
$h_{sk}(x_k) \leq \xi$, where $k = 1$ to i, and
$g_q(m_j) \leq 0$, where $q = 1$ to z.
z is the number of resource constraints. We then obtain AAC_i^*, and $m_j^{(i)}$, where $m_j^{(i)}$ is the optimal values of m_j for the first i intervals.
Step 3. Calculate $\mathrm{AAC}_{i+1}(m_j^{(i)})$, where $\mathrm{AAC}_{i+1}(m_j^{(i)})$, is the expected average annual cost of the $(i+1)$ intervals for the system configuration $m_j^{(i)}$.
Step 4. Is $\mathrm{AAC}_{i+1}(m_j^{(i)})$, greater than AAC_i^*? If yes, then stop. $m_j^{(i)}$ is the optimal system configuration with T_i as the economic life of the system. If no, the go to step 5.
Step 5. Set $i = i + 1$ and go to step 2.

The optimization problem at step 2 has nonlinear objective function with both linear and nonlinear constraints. We use GAs to solve the optimization problem. In Section 3 we describe the GAs and its advantages over traditional optimization methods and discuss the handling of constrained optimization problems using GAs.

3. Genetic algorithms

GAs are general purpose search and optimization techniques based on the mechanics of natural selection and genetics. They implement the survival of the fittest strategy among competing sets of coded parameters where each set represents a point in the domain of the search. At each iteration, or *generation* in GA terms, an initial population of parameter sets representing potential solutions to the search is generated. These parameter sets are encoded as strings of binary digits. At the start of the GA run, this population is generated by choosing points in the search domain at random. Each member of the population has associated with it a fitness value that depends on the coded parameters. This fitness can include both normal objective function values as well as penalties for constraint violations. Two operators, *selection* and *crossover*, are used to transform members of the initial population to form a new population. These processes are collectively known as *reproduction*: parents from the initial population produce offspring in the next generation. It is during the selection phase that the survival of the fittest strategy is implemented: parents that have a higher fitness are more likely to be selected to produce offspring. The process of *crossover* allows parents to exchange genetic information to form new offspring. Some of the resulting offspring will have a higher fitness than either parent; offspring with reduced fitness will be less likely to reproduce in subsequent generations. This is what ultimately drives the GA to reach the maximum possible fitness, i.e., the strong get stronger, and the weak die out. *Mutation* is an operator which perturbs the variable sets in some individuals in the population of offspring. In the classical binary coded GA parameter set, mutation involves flipping a binary digit to its opposite value. Although not the most important operator in the GA search, mutation can prevent populations from prematurely converging to sub-optimal solutions (genetic drift). The processes of reproduction and mutation are carried out repeatedly until either some convergence criterion is met or a maximum number of generations are reached. Convergence is typically assessed by comparing the average fitness of the population with the maximum currently present. For example, when the average fitness is within a few percent of the maximum, we can be assured that a large number of coded parameter sets representing the best solution are present in the population. Some of the many advantages of a GA apart from its easy adaptability and ability to solve diverse problems are:

- A GA searches from a population of potential solutions, rather than from a single point. This explicit parallelism helps the GA in determining close to global optimum solutions without the danger of being trapped in a local optimum. There is also an implicit parallelism (the Schema Theorem) that gives the GA its power in searching very large solution domains. Typically, a GA will be able to find a close-to-optimal solution by evaluating less than 1% of the points in the search space.
- A GA uses objective function information directly, with no need for derivatives or other information The objective function can involve any type of numeric or non-numeric variables, or other data structures, as long as a coding scheme can be devised to represent the parameter set.
- They are particularly suited to problems with a very large search space.
- They can treat with ease different types of variables (integer, real, non-numeric).

As mentioned earlier, in our formulation the objective function is nonlinear with both linear and non-linear constraints, and discrete variables. Nonlinear integer programming has been used to solve similar redundancy allocation problem with fixed component reliability by Tillman et al. (1985) and Misra and Sharma (1991).

A recent study by Coit and Smith (1996) has shown that GAs perform better than the traditional non-linear optimization techniques and can very efficiently solve a wide range of redundancy allocation problem. Monga et al. (1997) have demonstrated the use of GAs to solve a RBD problem with deteriorative components with an active failure rate constraint. In this formulation, we intend to generalize the RBD optimization problem by handling both active/non-active linear and non-linear constraints using GAs. To handle the non-active resource constraints of the form $g_q(m_j) \le 0$, where $q = 1, 2, 3, \ldots, z$, we incorporated the constraint violations as penalties (Goldberg, 1989) to the function we want to minimize.

$$F' = F + \sum_{q=1}^{z} P_q,$$

where F' the modified function to be minimized, F the original function to be minimized, z the number of constraints and P_q is the penalty for the violation of constraint q, such that

$$P_q = \begin{cases} 0 & \text{if} g_q(m_j) \leq 0 \\ |g_q(m_j)| & \text{otherwise} \end{cases}$$

This penalty method to handle constraints was tested successfully to solve reliability related non-linear optimization problems with discrete variables from (Tillman et al., 1985).

4. Numerical illustration

Consider a system with four subsystems connected in series. We need to determine the number of components in parallel at each subsystem. Assume all the components follow a Weibull distribution with shape parameter greater than one. The reliability functions of components in subsystems 1, 2, 3 and 4 are given by Eqs. (21)–(24), respectively.

$$r_1(t) = \mathrm{e}^{-0.5(t+0.008)^2} \quad (21)$$

$$r_2(t) = \mathrm{e}^{-0.15(t+0.005)^2} \quad (22)$$

$$r_3(t) = \mathrm{e}^{-0.055(t+0.006)^{1.5}} \quad (23)$$

$$r_4(t) = \mathrm{e}^{-0.095(t+0.003)^2} \quad (24)$$

Various costs associated with the system components are presented in Table 1. It can be seen that the component, which deteriorates faster costs less than the one which deteriorates more slowly. The installation costs of the system is 400 dollars and is independent of system design. The maximum allowed failure rate is 0.2 failures per year. The maximum number of components allowed in each sub system is equal to 15.

To solve the optimization problem a classical GA is implemented. For each subsystem j, the number of components m_j are coded as a 4-bit string. Each possible system design can then be described by a 16-bit string. For fitness evaluation for each coded string it is assumed that the failure rate constraint, i.e. Eq. (9) is active. An additional resource constraint is considered. This resource constraint in this

Table 1
Costs related to system components

j	AC_j	ϕ_j	MC_j	c_{mj}
1	90	1.11	10	1
2	125	1.2	15	1.5
3	150	1.33	20	2
4	225	1.11	25	2.5

formulation is referred as the *investment constraint*, and is given by Eq. (25).

$$\sum_{j=1}^{4} \phi_j \mathrm{AC}_j(1 + m_j) \leq 2500 \tag{25}$$

Since the GAs are set up to maximize fitness hence it is converted from a maximization problem into a minimization one. This is done by subtracting each string's AAC and constraint violation penalty from the largest in the current population to give a positive fitness value.

In the implementation of GAs, *linear scaling* is used to regulate the number of copies of extraordinary individuals within a population. Goldberg (1989) scaling routines: procedure *prescale*, function *scale* and procedure *scalepop* are used. *Stochastic remainder selection without replacement* is used as the selection method due to its superiority over other selection schemes. A linearly decreasing mutation rate is used over the first 40 generations with initial mutation probability as 0.05 and decreasing to 0 at the end of the 40th generation. Mutation in early generations helps the algorithm to fully explore the search space. However, later on during the run the creation of an individual via mutation generally will be the one with lower fitness of that population. Hence, it is not recommended to have mutation after 40 generations as it might be counter productive in obtaining a global optimum. A convergence criterion is set up to stop a run when the average fitness in the population is within 0.5% of the maximum fitness. The deterioration factors for each subsystem are given by Eqs. (26) and (27). According to these deterioration factors, the components in subsystems 1 and 4 deteriorate slower than the components in subsystems 2 and 3 after PM action.

$$\theta_{1i} = \theta_{4i} = 1 + \sum_{k=0}^{i-1} \left(\frac{k}{k+1} \right) \tag{26}$$

$$\theta_{2i} = \theta_{3i} = 1 + \sum_{k=0}^{i-1} \left(\frac{3k}{2k+1} \right) \tag{27}$$

The salvage value function defined by Eqs. (12) is used with

$$\rho = 2$$

$$\beta = 1.2$$

$$\Gamma_1 = 1,\ \Gamma_2 = 1.2,\ \Gamma_i = \Gamma_{i-1} + 0.1 \qquad \text{for} \qquad i = 3, 4, 5\ldots$$

To calculate the optimal system design for minimal life cycle cost we followed the *five step methodology* proposed by Monga et al. (1995). It is found that the optimal system design is 7, 3, 2 and 2 components in subsystem 1, 2, 3 and 4, respectively, (Table 2). The system should undergo PM at 1.227, 2.136 and 2.849 years. At the end of 3.420 years the system should not be maintained but replaced. The average annual cost of the system for its economic life is 526.785 dollars.

We then considered the same system with the same system characteristics but ignored the salvage value. The optimal design of 6, 3, 2 and 2 components at subsystem 1, 2, 3, and 4, respectively, is obtained. For such a system, it is found that PM should be performed at 1.165, 2.036, 2.714, 3.269,

Table 2
Implementation of *five step methodology* when salvage value is considered

I	AAC_i^*	$m_j^{(i)}+1$	$AAC_{i+1}(m_j^{(i)})$	$AAC_{i+1}(m_j^{(i)}) > AAC_i^*$	Action
1	765.113	7, 3, 1, 2	615.754	No	Next *i*
2	613.156	7, 3, 2, 2	545.016	No	Next *i*
3	545.016	7, 3, 2, 2	526.785	No	Next *i*
4	526.785	7, 3, 2, 2	528.679	Yes	Stop
5	528.679	7, 3, 2, 2	537.429	–	–
6	537.429	7, 3, 2, 2	578.898	–	–

3.738, 4.145, 4.507, 4.833 and 5.127 years. The system should be replaced at the end of 5.399 years. The expected average annual cost of the system over its economic life is 760.477 dollars (Table 3).

The economic life of the system is defined as the optimal length of time for which a system has a minimum average annual cost. In the case when the salvage value of the system is ignored, the net acquisition cost of the system is constant. The system cost increases with time due to increasing number of PM actions and minimal repairs. However, if the salvage value of the system is adjusted from the acquisition cost, the net acquisition cost of the system is no more a constant but increases with time during an operational period. The net acquisition cost of the system decreases right after the PM action is performed (due to increase in the salvage value) and then increases with a steeper slope than in the preceding interval. In such a situation when the economic life of a system is calculated at the end of each interval, there exists an economic incentive of capturing the decreasing salvage value of the system. This decreasing salvage value of the system pushes the system to be replaced earlier, hence making the economic life of the system shorter than the case when the salvage value is ignored. In addition, the minimal AAC is smaller.

Whether the salvage value of the system should be included in the cost function of a design problem will depend on the type of system one is considering. If one is designing a system, which has a custom defined function with a very limited market, then one is better off ignoring the salvage value of the

Table 3
Implementation of *Five Step Methodology* when salvage value is ignored

i	AAC_i^*	$m_j^{(i)}+1$	$AAC_{i+1}(m_j^{(i)})$	$AAC_{i+1}(m_j^{(i)}) > AAC_i^*$	Action
1	1996.055	7, 3, 2, 2	1241.570	No	Next *i*
2	1241.570	7, 3, 2, 2	1004.051	No	Next *i*
3	1004.051	7, 3, 2, 2	896.246	No	Next *i*
4	896.246	7, 3, 2, 2	837.207	No	Next *i*
5	837.207	7, 3, 2, 2	803.063	No	Next *i*
6	802.066	6, 3, 2, 2	781.953	No	Next *i*
7	781.953	6, 3, 2, 2	768.687	No	Next *i*
8	768.687	6, 3, 2, 2	762.664	No	Next *i*
9	762.664	6, 3, 2, 2	760.477	No	Next *i*
10	760.477	6, 3, 2, 2	761.527	Yes	Stop
11	761.527	6, 3, 2, 2	764.441	–	–
12	764.441	6, 3, 2, 2	769.324	–	–

system. The examples of such system are computer integrated manufacturing systems, which are custom designed to perform a very specific task like assembly of very expensive defense equipment. However, if there exists a market application for a system then there exists a salvage value for which the system can be sold readily at any given point of time. For such products, it is justified to include the salvage value at the design stage to incorporate the economic impact of salvage value on system design.

It must be recognized that system design is very sensitive to system failure characteristics and its relation to the salvage value function. When developing the salvage value function for a given system, the user should very carefully select various constants in the proposed salvage value function to reflect the actual salvage value of the system at a given point of time. For systems with short product life cycle, this would mean that the salvage value would be a very steep monotonically decreasing function. However for systems with long product life cycle the salvage value will decrease faster in the beginning and more slowly later on.

5. Concluding remarks

A system life cycle consists of four distinct phases: (1) system design, (2) system production, (3) system operation and maintenance, and (4) system retirement (Blanchard, 1992). In this paper we model the economic effects of all the four phases of a system life cycle by incorporating acquisition costs, preventive maintenance costs, costs due to minimal repairs and system's salvage value at the time of disposal. The model makes contribution towards effective economic evaluation of system design when the system follows a given salvage value function. The paper presents functions to model the effects of PM on the system's hazard rate and the salvage value as functions of time. The modified PM modeling incorporates both externally and internally induced failure rates to give a non-zero failure rate at time equal to zero. GAs are used as an optimization tool to optimize a nonlinear constrained cost function with discrete variables. The inclusion of the salvage value while evaluating a system design can be more of a strategic management decision than an engineering decision. However, these decisions will be more significant if the system components are very expensive and will have a resale value at any given point of time.

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