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ON THE BEHAVIOUR OF SOME NEW AGEING PROPERTIES BASED UPON THE RESIDUAL LIFE OF *k*-OUT-OF-*n* SYSTEMS

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Abstract

In this paper, we investigate k-out-of-n systems with independent and identically distributed components. Some characterizations of the IFR(2), DMRL, NBU(2) and NBUC classes of life distributions are obtained in terms of the monotonicity of the residual life given that the $(n-k)$th failure has occurred at time $t \geq 0$. These results complement those reported by Belzunce, Franco and Ruiz (1999). Similar conclusions based on the residual life of a parallel system conditioned by the $(n-k)$th failure time are presented as well.

Keywords: DMRL; IFR(2); NBUC; NBU(2); NBU_{Lt}; increasing convex order; increasing concave order; parallel system; residual life

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1. Introduction and preliminaries

In reliability theory, nonparametric ageing classes of life distributions, such as IFR (increasing failure rate), NBU (new better than used) and NBUE (new better than used in expectation), have been found to be quite useful in maintenance optimization and system analysis. As a result, an extensive list of related research results have been achieved over the past decades. Several extensions of these notions, for example, IFR(2) (increasing failure rate in second stochastic dominance), DMRL (decreasing mean residual life), NBU(2) (new better than used in second stochastic dominance), NBUC (new better than used in convex ordering), and NBU_{Lt} (new better than used in Laplace order), have been reported in the literature as well. These ageing notions often stress that the lifetime of a brand new element is larger in some stochastic sense than the residual life of a used one. Many authors have paid attention to the behaviour of ageing properties in coherent structures such as parallel and series systems and the k-out-of-n systems. They are also studied under convolution, mixture and the renewal process. For more details, we refer the reader to Barlow and Proschan (1981), Langberg *et al.* (1980), Hendi *et al.* (1993), Chen (1994), Belzunce *et al.* (1999), Li *et al.* (2000), Pellerey and Petakos (2002), Li and Kochar (2001), Franco *et al.* (2001) and Belzunce *et al.* (2001).

In this paper, we present some new results on the behaviours of IFR(2), NBU(2), DMRL and NBUC ageing properties in terms of the residual life of a k-out-of-n system with i.i.d. components conditioned by the time of the $(n-k)$th failure. We also give similar ageing property results for parallel systems.

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Before proceeding to state our main results, we first give an overview of several criteria for comparison of random variables. These criteria are widely used in describing ageing notions and making risk analysis.

In what follows:

- X and Y are nonnegative random variables, representing equipment lives,
- F and G are the cumulative distribution functions (CDFs) of X and Y respectively,
- $\bar{F}$ and $\bar{G}$ are the survival functions of X and Y respectively, so $\bar{F} = 1 - F$ and $\bar{G} = 1 - G$,
- F^{-1} and G^{-1} are the right-continuous inverses of the CDFs F and G respectively.

Definition 1. (Shaked and Shanthikumar (1994).)

(a) We say that Y is larger than X in increasing convex order (denoted by $X \leq_{\text{icx}} Y$) if $\mathrm{E}\, h(X) \leq \mathrm{E}\, h(Y) < \infty$ for all increasing and convex functions h.

(b) We say that Y is larger than X in increasing concave order (denoted by $X \leq_{\text{icv}} Y$) if $\mathrm{E}\, h(X) \leq \mathrm{E}\, h(Y) < \infty$ for all increasing and concave functions h.

(c) We say that Y is larger than X in Laplace transform order (denoted by $X \leq_{\text{Lt}} Y$) if $\mathrm{E}\, \mathrm{e}^{-sY} \leq \mathrm{E}\, \mathrm{e}^{-sX}$ for all $s \geq 0$.

Shaked and Shanthikumar (1994) and Stoyan (1983) have shown that $X \leq_{\text{icx}} Y$ if and only if, for all $t \geq 0$,

$$\int_t^\infty \bar{F}(x)\,\mathrm{d}x \leq \int_t^\infty \bar{G}(x)\,\mathrm{d}x,$$

and $X \leq_{\text{icv}} Y$ if and only if, for all $t \geq 0$,

$$\int_0^t \bar{F}(x)\,\mathrm{d}x \leq \int_0^t \bar{G}(x)\,\mathrm{d}x.$$

In addition, the usual stochastic order is sufficient for both the increasing convex order and the increasing concave order. For a more comprehensive discussion on properties and other details of those stochastic orderings, we refer the reader to Shaked and Shanthikumar (1994) and Stoyan (1983).

In the following, we give definitions of some ageing properties that are closely related to the main results to be presented in this paper. For details of the results to be summarized here, see Hollander and Proschan (1984), Deshpande *et al.* (1986), and Cao and Wang (1991).

Let $X_t = (X - t \mid X > t)$ be the residual life at age $t > 0$ of the random life X.

Definition 2. (Hollander and Proschan (1984), Deshpande *et al.* (1986), Cao and Wang (1991).)

(a) We say that X is IFR (increasing failure rate) if X_t is stochastically decreasing in $t \geq 0$.

(b) We say that X is DMRL (decreasing mean residual life) if $\mathrm{E}\, X_t$ is decreasing in $t \geq 0$.

(c) We say that X is IFR(2) (increasing failure rate of second-order stochastic dominance) if X_t is stochastically decreasing in $t \geq 0$ in increasing concave order.

(d) We say that X is NBU (new better than used) if $X_t \leq_{\text{st}} X$ for all $t \geq 0$.

(e) We say that X is NBU(2) (new better than used of second-order stochastic dominance) if $X_t \leq_{\text{icv}} X$ for all $t \geq 0$, or equivalently

$$\int_0^t \bar{F}(y+x)\,\mathrm{d}x \leq \bar{F}(y) \int_0^t \bar{F}(x)\,\mathrm{d}x \quad \text{for all } t, y \geq 0. \tag{1}$$

(f) We say that X is NBUC (new better than used in convex order) if $X_t \leq_{\text{icv}} X$ for all $t \geq 0$, or equivalently

$$\int_t^\infty \bar{F}(y+x)\,\mathrm{d}x \leq \bar{F}(y) \int_t^\infty \bar{F}(x)\,\mathrm{d}x \quad \text{for all } t, y \geq 0. \tag{2}$$

(g) We say that X is NBU_{Lt} (new better than used in Laplace order) if $X_t \leq_{\text{Lt}} X$ for all $t \geq 0$.

Cao and Wang (1991, Theorem 2.4) prove that X is DMRL if and only if X_t is decreasing in $t \geq 0$ in increasing convex order. As the dual notions, IMRL (increasing mean residual life), DFR(2) (decreasing failure rate of second-order stochastic dominance), NWU(2) (new worse than used of second-order stochastic dominance), NWUC (new worse than used in convex order) and NWU_{Lt} (new worse than used in Laplace order) can be defined by reversing the monotonicity and inequalities accordingly.

The following chain of implications can easily be established:

$$\begin{array}{ccccc} & \text{DMRL} & \Longrightarrow & \text{NBUC} & \\ \nearrow & & & & \searrow \\ \text{IFR} & & & & \text{NBUE} \\ \searrow & & & & \nearrow \\ & \text{IFR(2)} & \Longrightarrow & \text{NBU(2)} & \end{array}$$

For completeness and clarity, we now present some related results on ageing notions in the above chain of implications.

A k-out-of-n system works if and only if at least k components work or, equivalently, at most $n-k$ elements fail. Thus, the life of a k-out-of-n system can be characterized by the $(n-k+1)$th order statistic $X_{n-k+1,n}$. Here, $X_{i,n}$ represents the ith smallest failure time among n failure times. The series system and the parallel system can be considered special cases of the k-out-of-n systems. A series system is an n-out-of-n system and a parallel system is a 1-out-of-n system. Given that the $(n-k)$th failure occurs at time $t \geq 0$, the residual life of a k-out-of-n system is represented by the conditional random variable,

$$RLS_{k,n,t} = (X_{n-k+1,n} - X_{n-k,n} \mid X_{n-k,n} = t) = \min_{1 \leq i \leq k} (X_i)_t,$$

where $(X_1)_t, \ldots, (X_k)_t$ are i.i.d. copies of X_t. With these definitions, the life of the series system with k components can be expressed as

$$LS_k = X_{1,k} = \min\{X_1, \ldots, X_k\}.$$

The survival functions of a k-out-of-n system and a series system can be expressed as

$$\Pr(RLS_{k,n,t} > x) = \left[\frac{\bar{F}(x+t)}{\bar{F}(t)}\right]^k = \bar{F}_t^k(x), \qquad x \geq 0, \tag{3}$$

and

$$\Pr(LS_k > x) = \bar{F}^k(x), \qquad x \geq 0.$$

To investigate how the failure time of a component affects the lifetime of a k-out-of-n system, Langberg *et al.* (1980) present the following characterizations for the IFR and NBU notions and their duals:

$$\begin{aligned} X \text{ is IFR} &\iff RLS_{k,n,t'} \geq_{\text{st}} RLS_{k,n,t}, \\ X \text{ is DFR} &\iff RLS_{k,n,t'} \leq_{\text{st}} RLS_{k,n,t}, \\ X \text{ is NBU} &\iff LS_k \geq_{\text{st}} RLS_{k,n,t}, \\ X \text{ is NWU} &\iff LS_k \leq_{\text{st}} RLS_{k,n,t}, \end{aligned}$$

for all $t \geq t' \geq 0$ and for any integer k such that $1 \leq k < n$. Recently, Belzunce *et al.* (1999) provided additional results on other stochastic orders. In addition to some equivalent characterizations of IFR in terms of the disperse order and failure rate order, they also propose the following characterization of the DMRL property of a k-out-of-n system, which is denoted by DMRLS, and its dual, IMRLS:

$$\begin{aligned} X \text{ is DMRLS} &\iff RLS_{k,n,t'} \geq_{\text{icx}} RLS_{k,n,t} \text{ for all } t \geq t' \geq 0, \\ X \text{ is IMRLS} &\iff RLS_{k,n,t'} \leq_{\text{icx}} RLS_{k,n,t} \text{ for all } t \geq t' \geq 0. \end{aligned} \tag{4}$$

Since DMRLS (that is, $\mathrm{E}(RLS_{k,n,t})$ is decreasing in $t \geq 0$ for all $1 \leq k < n$) implies NBUES (that is, $\mathrm{E}(RLS_{k,n,t}) \leq \mathrm{E}(LS_k)$ for all $t \geq 0$ and $1 \leq k < n$), the right-hand side of (4) can be treated as a sufficient condition for NBUES. Although Belzunce *et al.* (1999) state that $LS_k \geq_{\text{icx}} RLS_{k,n,t}$ does not characterize any other classes discussed earlier, in light of the relations among NBU, NBUC, NBU(2) and DMRL, we may ask whether

$$LS_k \geq_{\text{icx}} RLS_{k,n,t} \quad \text{or} \quad LS_k \leq_{\text{icx}} RLS_{k,n,t}$$

is sufficient for NBUC and whether

$$LS_k \geq_{\text{icv}} RLS_{k,n,t} \quad \text{or} \quad LS_k \leq_{\text{icv}} RLS_{k,n,t}$$

is implied by NBU(2). These two conjectures will be proved to be valid in Theorem 1 below.

In this paper, we pay special attention to the residual life of a 1-out-of-n (parallel) system given that the $(n-r)$th failure occurs at time $t \geq 0$,

$$RLP_{r,n,t} = (X_{n,n} - X_{n-r,n} \mid X_{n-r,n} = t) = \max_{1 \leq i \leq r} (X_i)_t,$$

and the life span of a parallel system with r components,

$$LP_r = X_{r,r} = \max\{X_1, \dots, X_r\}.$$

The survival functions of these parallel and series systems should be, for all $x \geq 0$,

$$\mathrm{P}(X_{n,n} - X_{n-r,n} > x \mid X_{n-r,n} = t) = 1 - F_t^r(x),$$

where

$$F_t(x) = 1 - \bar{F}_t(x) = 1 - \frac{\bar{F}(t) - \bar{F}(x+t)}{\bar{F}(t)}$$

and

$$\mathrm{P}(LP_r > x) = 1 - F^r(x).$$

Properties of NBU(2) and NBUC in terms of $RLP_{r,n,t}$ and LP_r, which are analogous to Theorem 1, will be provided in Theorem 3. Furthermore, Theorems 2 and 4 give results on IFR(2) and DMRL dual to Theorems 1 and 3.

For ease of reference, the term *increasing* is used instead of *monotone nondecreasing* and the term *decreasing* is used instead of *monotone nonincreasing* throughout this paper. We also assume that the random variables under consideration have 0 as the common left end point of their supports, and the expectation is assumed to be finite when used.

2. Main results

The following lemma is provided here for ease of reference.

Lemma 1. (Barlow and Proschan (1981, p. 120).) *Let W be a measure on the interval (a, b), not necessarily nonnegative, and h a nonnegative function defined on (a, b).*

(a) *If $\int_t^b \mathrm{d}W(x) \geq 0$ for all $t \in (a, b)$ and if h is increasing, then $\int_a^b h(x)\,\mathrm{d}W(x) \geq 0$.*

(b) *If $\int_a^t \mathrm{d}W(x) \geq 0$ for all $t \in (a, b)$ and if h is decreasing, then $\int_a^b h(x)\,\mathrm{d}W(x) \geq 0$.*

As our first main result, Theorem 1 describes the behaviour of NBU(2) and NBUC in terms of the conditional residual life of a k-out-of-n system.

Theorem 1. *Let k be an integer such that $1 \leq k < n$.*

(i) *If X is NBU(2), then $LS_k \geq_{\mathrm{icv}} RLS_{k,n,t}$ for all $t \geq 0$, and if X is NWU(2), then $LS_k \leq_{\mathrm{icv}} RLS_{k,n,t}$ for all $t \geq 0$.*

(ii) *If $LS_k \geq_{\mathrm{icx}} RLS_{k,n,t}$ for all $t \geq 0$, then X is NBUC, and if $LS_k \leq_{\mathrm{icx}} RLS_{k,n,t}$ for all $t \geq 0$, then X is NWUC.*

Proof. Denote by F and F_t the CDFs of X and X_t respectively, and by $\bar{F}$ and $\bar{F}_t$ the corresponding survival functions. We prove (i) for the NBU(2) case and (ii) for the NBUC case; the results for NWU(2) and NWUC follow by reversing the inequalities.

(i) Suppose that X is NBU(2), then, by (1),

$$\int_0^y \left[\bar{F}(x) - \bar{F}_t(x)\right] \mathrm{d}x \geq 0 \quad \text{for all } y \geq 0 \text{ and } t \geq 0.$$

For any $t \geq 0$, the function

$$\bar{F}^{k-1}(x) + \bar{F}^{k-2}(x)\bar{F}_t(x) + \cdots + \bar{F}(x)\bar{F}_t^{k-2}(x) + \bar{F}_t^{k-1}(x)$$

is nonnegative and decreasing. It follows from Lemma 1(b) that

$$\int_0^y \left[\bar{F}^k(x) - \bar{F}_t^k(x)\right] \mathrm{d}x \geq 0 \quad \text{for all } y \geq 0 \text{ and } t \geq 0.$$

Note that LS_k has its own survival function, $\bar{F}^k(x)$. By (3), this inequality gives the desired result directly.

(ii) Notice that $LS_k \geq_{\mathrm{icx}} RLS_{k,n,t}$ for all $t \geq 0$ implies that

$$\int_y^\infty \left[\bar{F}^k(x) - \bar{F}_t^k(x)\right] \mathrm{d}x \geq 0 \quad \text{for all } t \geq 0 \text{ and } y \geq 0,$$

and

$$\left[\bar{F}^{k-1}(x)+\bar{F}^{k-2}(x)\bar{F}_t(x)+\cdots+\bar{F}(x)\bar{F}_t^{k-2}(x)+\bar{F}_t^{k-1}(x)\right]^{-1}$$

is increasing and nonnegative. The NBUC property of X can be derived from Lemma 1(a) in a similar way.

The following theorem describes similar properties of IFR(2) and DMRL.

Theorem 2. *Let k be an integer such that $1 \le k < n$.*

(i) *If X is IFR(2), then $RLS_{k,n,t'} \ge_{\text{icv}} RLS_{k,n,t}$ for $t \ge t' \ge 0$, and if X is DFR(2), then $RLS_{k,n,t'} \le_{\text{icv}} RLS_{k,n,t}$ for $t \ge t' \ge 0$.*

(ii) *If $RLS_{k,n,t'} \ge_{\text{icx}} RLS_{k,n,t}$ for $t \ge t' \ge 0$, then X is DMRL, and if $RLS_{k,n,t'} \le_{\text{icx}} RLS_{k,n,t}$ for $t \ge t' \ge 0$, then X is IMRL.*

Proof. Note that the IFR(2) property of X implies that, for all $t \ge t' \ge 0$ and $y \ge 0$,

$$\int_0^y \bar{F}_t(x)\,\mathrm{d}x \le \int_0^y \bar{F}_{t'}(x)\,\mathrm{d}x,$$

and, by (4), a k-out-of-n system is DMRL if and only if, for $t \ge t' \ge 0$, $y \ge 0$ and an integer k such that $1 \le k \le n$,

$$\int_y^\infty \bar{F}_t^k(x)\,\mathrm{d}x \le \int_y^\infty \bar{F}_{t'}^k(x)\,\mathrm{d}x.$$

These two results can then be concluded in a similar manner to the proof of Theorem 1.

Remark 1. It was pointed out by Belzunce *et al.* (1999, Remark 3.1) that DMRL does not imply DMRLS. However, as stated in Theorem 2, the opposite implication is true. In fact, we have

(i) if X is IFR(2), then X is IFR(2)S, and if X is DFR(2), then X is DFR(2)S,

(ii) if X is DMRLS, then X is DMRL, and if X is IMRLS, then X is IMRL.

Here X is said to be IFR(2)S if, for any k such that $1 \le k < n$, the residual life $RLS_{k,n,t}$ is decreasing in $t \ge 0$ in terms of increasing concave order, and is said to be DFR(2)S if it is increasing in $t \ge 0$.

The following theorem describes the behaviour of NBU(2) and NBUC in terms of the residual life of a parallel system conditioned by the $(n-r)$th failure time.

Theorem 3. *Let r be an integer such that $0 \le r < n$.*

(i) *If $LP_r \ge_{\text{icv}} RLP_{r,n,t}$ for all $t \ge 0$, then X is NBU(2), and if $LP_r \le_{\text{icv}} RLP_{r,n,t}$ for all $t \ge 0$, then X is NWU(2).*

(ii) *If X is NBUC, then $LP_r \ge_{\text{icx}} RLP_{r,n,t}$ for all $t \ge 0$, and if X is NWUC, then $LP_r \le_{\text{icx}} RLP_{r,n,t}$ for all $t \ge 0$.*

Proof. (i) If $LP_r \ge_{\text{icv}} RLP_{r,n,t}$, then, for all $y \ge 0$ and $t \ge 0$,

$$\int_0^y [\mathrm{P}(LP_r > x) - \mathrm{P}(RLP_{r,n,t} > x)]\,\mathrm{d}x = \int_0^y [F_t^r(x) - F^r(x)]\,\mathrm{d}x \ge 0.$$

Note that, for all $t \ge 0$,

$$\left[F^{r-1}(x)+F^{r-2}(x)F_t(x)+\cdots+F(x)F_t^{r-2}(x)+F_t^{r-1}(x)\right]^{-1}$$

is nonnegative and decreasing. From Lemma 1(b), for all $y \geq 0$ and $t \geq 0$, we have

$$\int_0^y \left[\bar{F}(x) - \bar{F}_t(x)\right] \mathrm{d}x = \int_0^y [F_t(x) - F(x)] \, \mathrm{d}x \geq 0.$$

As a result, X is NBU(2).

(ii) By (2), the NBUC property implies that, for all $y \geq 0$ and $t \geq 0$,

$$\int_y^\infty \left[\bar{F}(x) - \bar{F}_t(x)\right] \mathrm{d}x = \int_y^\infty [F_t(x) - F(x)] \, \mathrm{d}x \geq 0.$$

Since, for any $t \geq 0$, the function

$$F^{r-1}(x) + F^{r-2}(x)F_t(x) + \cdots + F(x)F_t^{r-2}(x) + F_t^{r-1}(x)$$

is nonnegative and increasing, it follows from Lemma 1(a) that, for all $y \geq 0$ and $t \geq 0$,

$$\int_y^\infty [\mathrm{P}(LP_r > x) - \mathrm{P}(RLP_{r,n,t} > x)] \, \mathrm{d}x = \int_y^\infty [F_t^r(x) - F^r(x)] \, \mathrm{d}x \geq 0.$$

Thus, $LP_r \geq_{\mathrm{icx}} RLP_{r,n,t}$ for all $t \geq 0$.

For the cases of NWU(2) and NWUC it suffices to reverse the inequalities in the above statements.

Parallel to Theorem 2, we have the following result; its proof is routine and is thus omitted.

Theorem 4. *Let r be an integer such that $0 \leq r < n$.*

(i) *If $RLP_{r,n,t'} \geq_{\mathrm{icv}} RLP_{r,n,t}$ for $t \geq t' \geq 0$, then X is IFR(2), and if $RLP_{r,n,t'} \leq_{\mathrm{icv}} RLP_{r,n,t}$ for $t \geq t' \geq 0$, then X is DFR(2).*

(ii) *If X is DMRL, then $RLP_{r,n,t'} \geq_{\mathrm{icx}} RLP_{r,n,t}$ for $t \geq t' \geq 0$, and if X is IMRL, then $RLP_{r,n,t'} \leq_{\mathrm{icx}} RLP_{r,n,t}$ for $t \geq t' \geq 0$.*

Remark 2. (i) According to Cao and Wang (1992),

$$\text{DMRL} \implies \text{NBUC} \implies \text{NBUE}.$$

Thus, the first counterexample in Remark 3.1 of Belzunce *et al.* (1999) also reveals that

$$\text{NBUC} \not\Longrightarrow \text{NBUCS}.$$

Here X is said to be NBUCS if, for any k such that $1 \leq k < n$ and $t \geq 0$, $RLS_{k,n,t}$ is smaller than LS_k in terms of increasing convex order.

(ii) Although

$$\text{NBU(2)} \implies \text{NBU}_{\mathrm{Lt}} \implies \text{NBUE},$$

by Theorem 3.2 of Belzunce *et al.* (2001) it can be proved that, if the common survival function of all components is completely monotone, then X is NBU_{Lt} implies that $LS_k \geq_{\mathrm{Lt}} RLS_{k,n,t}$ for all $t \geq 0$ and k such that $1 \leq k < n$, and if X is NWU_{Lt}, then $LS_k \leq_{\mathrm{Lt}} RLS_{k,n,t}$ for all $t \geq 0$ and k such that $1 \leq k < n$.

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