Extraction of Periodic Components for Gearbox Diagnosis Combining Wavelet Filtering and Cyclostationary Analysis

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Wavelet filtering is combined with cyclostationary analysis for detection of gear tooth faults in a gearbox. The parameters of the wavelet filter are optimized by using the proposed entropy minimization rule. This method is shown to be effective in detecting gear faults when cyclostationary analysis by itself fails. [DOI: 10.1115/1.1760565]

1 Introduction

Wavelet analysis and cyclostationary analysis have both been employed to detect fault symptoms in gearboxes [1–5]. For periodic impulse detection, the Morlet wavelet was employed due to its similarity to an impulse [3]. Randall et al. [4] used cyclostationary analysis to diagnose roller bearings. Capdessus et al. [5] used this method to detect early faults in gearboxes. It was demonstrated that the vibration signals were cyclostationary when faults were present in the monitored equipment [4,5].

When the background noise is heavy, it is difficult to disclose the latent periodic components successfully using cyclostationary analysis alone, due to the low signal to noise ratio (SNR). Preprocessing needs to be performed to increase SNR in order to enhance the effectiveness of cyclostationary analysis. In this paper, we describe the use of a wavelet filter as a preprocessing tool to help find the cyclostationarity of signals.

2 Cyclostationary Analysis

A random signal $x(t)$ is said to be cyclostationary at the nth order if its time domain nth order moment is a periodic function of time t [5]. Second order cyclostationarity is often used because it is sensitive to transient mechanical faults. The signal $x(t)$ is cyclostationary at the second order if the following equation is satisfied,

$$R_x(t,\tau)=E[x(t+\tau/2)x(t-\tau/2)]=R_x(t+T,\tau), \qquad (1)$$

where E indicates the mathematical expectation and T is the cyclic period. The cyclic frequency is represented by α, $\alpha=1/T$.

The degree of cyclostationary (DCS) was proposed by Zivanovic and Gardner [6] as a quantitative description of cyclostationarity. It measures the amount of energy in the signals that is due to the cyclostationary components at frequency α. The discrete version of the definition of DCS is

$$\mathrm{DCS}(\alpha)=\frac{\sum_{\tau}|R_x^{\alpha}(\tau)|^2}{\sum_{\tau}|R_x^{0}(\tau)|^2}. \qquad (2)$$

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It can also be expressed in the frequency domain as

$$\mathrm{DCS}(\alpha)=\frac{\sum_{f}|S_x^{\alpha}(f)|^2}{\sum_{f}|S_x^{0}(f)|^2}, \qquad (3)$$

where $S_x^{\alpha}(f)$ is the Fourier transform of $R_x^{\alpha}(\tau)$ with respect to τ. $S_x^{\alpha}(f)$ can be expressed as

$$S_x^{\alpha}(f)=E[X(f-\alpha/2)X^*(f+\alpha/2)], \qquad (4)$$

where $X(f)$ is the Fourier transform of signal $x(t)$ and the symbol* stands for complex conjugation. We can express $X(f-\alpha/2)$ as

$$X(f-\alpha/2)=\int_{-\infty}^{\infty}x(t)e^{\pi\alpha jt}e^{-2\pi jft}dt. \qquad (5)$$

The DCS may take values in the range of 0 to 1. For $\alpha\neq 0$, the larger the DCS value, the more cyclostationary components there are in the signal.

3 Wavelet Filters

Wavelet transforms are inner products of the signals and the wavelet family, which is derived from the mother wavelet through dilation and translation. Let $\psi(t)$ be the mother wavelet, the daughter wavelet will be $\psi_{a,b}(t)=\psi(t-b/a)$, where a is the scale parameter and b is the time translation. By varying the parameters a and b, we can obtain different daughter wavelets that constitute a wavelet family. If a daughter wavelet is viewed as a filter, wavelet transform is actually parallel band-pass filtering. We can make the wavelet filter adapt to the signal to get better filtering performance. The Morlet wavelet was shown to be effective for extracting impulses in signals [3]. The Morlet wavelet is defined as

$$\psi(t)=e^{-\beta^2t^2/2}e^{-j\pi t}. \qquad (6)$$

Usually, the real part of the definition is taken as the basic wavelet, that is,

$$\psi(t)=e^{-\beta^2t^2/2}\cos(\pi t). \qquad (7)$$

The daughter wavelets can be obtained by performing scale dilation and time translation:

$$\psi_{a,b}(t)=\psi\left(\frac{t-b}{a}\right)=\exp\left[-\frac{\beta^2(t-b)^2}{2a^2}\right]\cos\left[\frac{\pi(t-b)}{a}\right]. \qquad (8)$$

Because there is no down sampling in the calculation, the step size of the parameter b is set equal to the time duration between adjacent data points in the original data series. Thus, only parameters a and β in formula (8) need to be adjusted. As a result, we define a Morlet wavelet filter as

$$\psi_a(t)=\psi\left(\frac{t}{a}\right)=\exp\left[-\frac{\beta^2t^2}{2a^2}\right]\cos\left[\frac{\pi t}{a}\right]. \qquad (9)$$

Entropy is a concept used in information theory [7]. The entropy for a random probability distribution $P=\{p_1,p_2,\cdots,p_N\}$ can be calculated as

$$En=-\sum_{i=1}^{N}p_i\ln p_i. \qquad (10)$$

Based on this equation, a uniform distribution has the largest entropy value. For impulses, the distribution of the time series is ‘‘sparse,’’ which means that there is a spike at the zero position of the probability distribution and the amplitude decreases quickly as time goes on. The corresponding entropy of an impulse is much smaller than that of a uniform distribution. The impulse components in the filtered results should be as prominent as possible. The corresponding entropy value should be as small as possible.

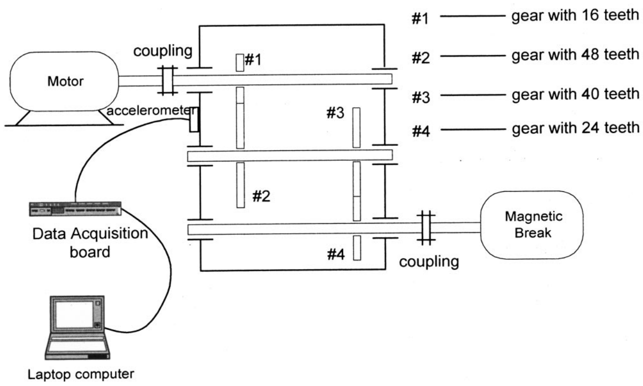

Fig. 1 Diagram of the experimental system

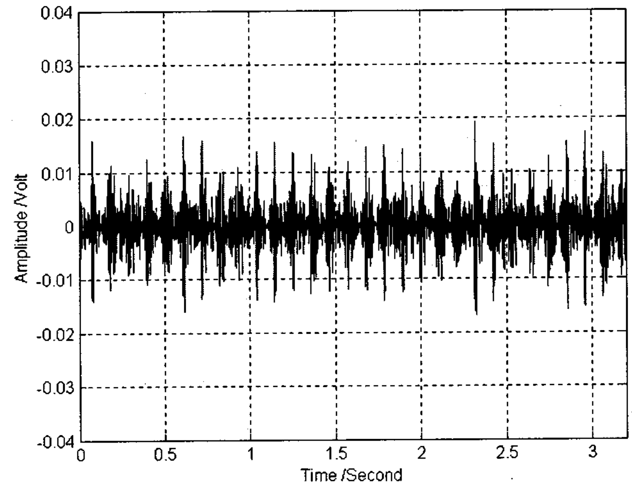

Fig. 3 Signals obtained with wavelet filter from the signals with one damaged tooth on gear #1

As a result, we propose to use entropy minimization as the guide for choosing the best values of the parameters a and β of the wavelet filter.

4 Diagnosing a Gearbox Using a Wavelet Filter and Cyclostationary Analysis

An experiment was conducted on a gearbox dynamic simulator. The diagram of the experimental system is illustrated in Fig. 1. Spur gears were used in the gearbox. We collected signals from the gearbox with one damaged tooth on gear #1. The damage was generated by chipping off part of the tooth. The signals were collected through an accelerometer located on the input shaft side of the gearbox. The sampling frequency used was 2560 Hz. The number of data points collected was 8192. The rotating frequency of the motor was set at 9 Hz, which was also the rotating frequency of gear #1. The meshing frequency of gear #1 was 144 Hz.

When a tooth is chipped, the stiffness of the tooth changes. Relative to the impulses generated by normal teeth, the one generated by the chipped tooth is expected to stand out as different. In the following, wavelet filtering and DCS plots are used in combination to isolate the impulses in the collected signals.

Figure 2 shows the signal waveform from the gearbox with one damaged tooth on gear #1. Morlet wavelet filtering was applied to this data series. The parameters of the filter were determined according to the entropy minimization rule as described in Section 2. The signals purified with the optimal wavelet filter are shown in Fig. 3.

We then used the DCS plots to find the period of latent impulses caused by the damaged tooth. The DCS plot is shown in Fig. 4. The largest DCS value in Fig. 4 corresponds to the cyclic frequency of 9 Hz, which was exactly the rotating frequency of gear #1.

For comparison, the DCS plot of the original signal, which was not processed with the wavelet filter, is given in Fig. 5. Almost all the DCS values are around or less than 0.01, which is much smaller than the DCS value of the signals obtained with the wavelet filter. As shown in Fig. 4, many of the DCS values of the filtered signals were higher than 0.15. The DCS value corresponding to the rotating frequency of the damaged gear was 0.41.

This experiment has shown that combining wavelet filtering and cyclostationary analysis can reveal latent periodic impulses more effectively than cyclostationary analysis alone.

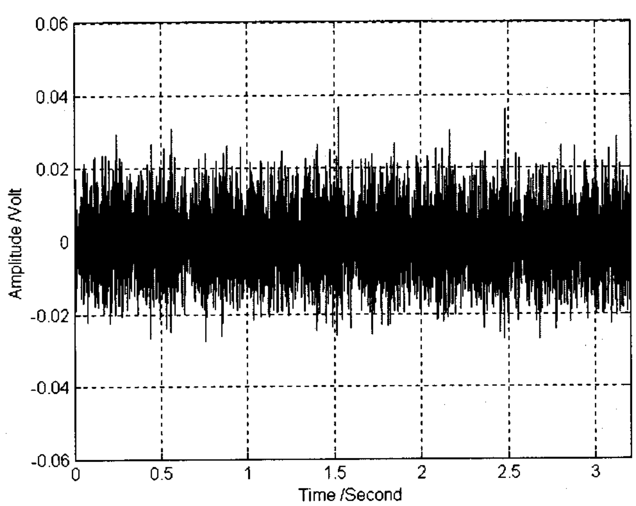

Fig. 2 Signals from the gearbox with one damaged tooth on gear #1

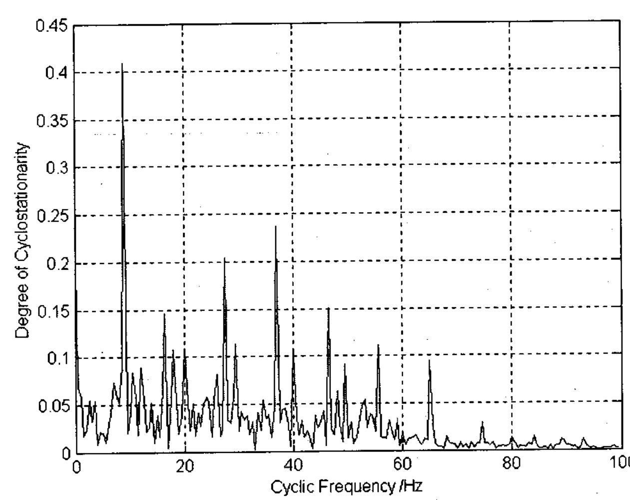

Fig. 4 DCS plot of the purified signals with one damaged tooth on gear #1

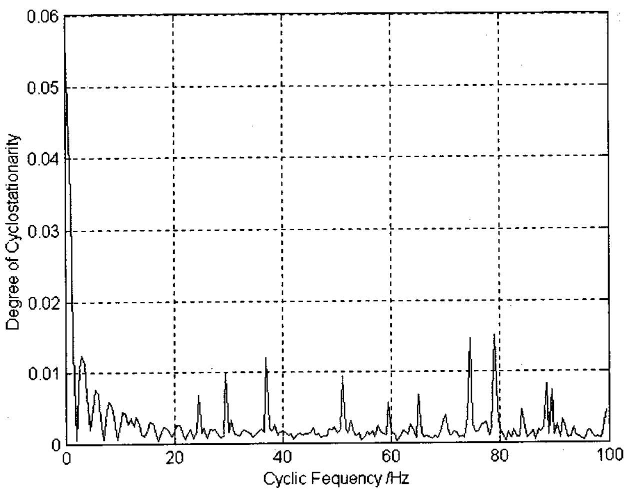

Fig. 5 DCS plot of the noisy signals with one damaged tooth on gear #1

5 Conclusion

For signals with heavy noise, DCS by itself may not be able to discover latent cyclic impulses. A daughter Morlet wavelet is introduced as a filter to remove the noise. We have demonstrated that, when the proposed entropy minimization principle for filter parameter determination is used, the filter can effectively extract the impulses from the noisy signals. For the filtered signals, the period of the impulses can then be disclosed using the DCS plots. Using wavelet filtering and cyclostationary analysis in combination is an efficient way of detecting latent periodic impulses in noisy signals.

Acknowledgment

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